St. Michael's School

Jajpur, Soparom, Ranchi-835303, Jharkhand.
(An English Medium Co-Educational School - Affiliated to C.B.S.E, Bharat)

Xth

Practice Sample Papers

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A complete assessment prepared as per the latest syllabus issued by C.B.S.E, Bharat.

MATHEMATICS

REAL NUMBERS

1. State:
   (a) Euclid's Division Lemma **(b)** Fundamental Theorem of Arithmetic.
2. A sweet seller has 420 *kaju barfis* and 130 *badam barfis* . She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the number of that can be placed in each stack for this purpose?
3. Use Euclid's division algorithm to find the HCF of 196 and 38220.
4. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?
5. If n is a positive integer, show that $(n^2 - 1)$ is divisible by 8.
6. S how that every positive odd integer is of the form $(6q + 1)$ or $(6q + 3)$ or $(6q + 5)$ for some integer q.
7. Show that one and only one out of n, $(n + 1)$ and $(n + 2)$ is divisible by 3, where n is any positive integer.
8. Show that one and only one out of $n, n + 2, n + 4$ is divisible by 3, where n is any positive integer.
9. Use Euclid's Division Lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m.
10. Use Euclid's Division Lemma, show that the cube of any positive integer is of the form $9q$ or $(9q + 1)$ or $(9q + 8)$ for some integer q.
11. Show that any number of the form 6^n, where $n \in N$ can never end with the digit 0.
12. Find the largest number which divides 546 and 764, leaving remainders 6 and 8 respectively.
13. By what number should 1365 be divided to get 31 as quotient and 32 as remainder?
14. Three pieces of timber 42m, 49m and 63m long have to be divided into planks of the same length. What is the greatest possible length of each plank?
15. Three sets of English, Mathematics and Science books containing 336, 240 and 96 books respectively have to be stacked in such a way that all the books are stored subjects wise and the height of each stack is the same. How many stacks will be there?
16. A electronic device makes a beep after every 60 seconds. Another device makes a beep after every 62 seconds. They beeped together at 10 am. At what time will they beep together at the earliest?
17. Six bells commence tolling together and toll at intervals of 2, 4, 6, 8, 10, 12 minutes respectively. In 30 hours, how many times do they toll together?
18. There is circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same direction. Suppose they both start at the same point and at the same time and go in the same direction. After how many minutes will they meet again at the starting point?
19. Find the HCF and LCM of 6, 72 and 120, using the prime factorisation method.
20. Express the number 7429 as a product of prime factors.

21. Given that HCF (306, 657)=9, find LCM (306, 657).

22. Explain why 7 X 11 X 13 + 13 and 7 X 6 X 5 X 4 X 3 X 2 X 1 + 5 composite numbers.

23. The decimal expansion of the rational number $\frac{43}{2^4.5^3}$, will terminate after how many places of decimals.

24. Prove that of the following irrational numbers:
(a) $(3+5\sqrt{2})$ **(b)** $(5-2\sqrt{3})$ **(c)** $\sqrt{5}$ **(d)** $(\sqrt{2}+\sqrt{3})$

25. Write a rational number between $\sqrt{2}$ and $\sqrt{3}$.

26. If 'd' is the HCF of 56 and 72. Find x and y satisfying d= $56x + 72y$. Also, show that x and y are not unique.

QUADRATIC EQUATIONS

27. **Solve the following equations:**

a) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}$

b) $\frac{x-1}{x-2} + \frac{x-3}{x-4} = \frac{10}{3}$

c) $\frac{1}{x+3} + \frac{1}{2x-1} = \frac{11}{7x+3}$

d) $2\left(\frac{2x-1}{x+3}\right) - 3\left(\frac{x+3}{2x-1}\right) = 5$

e) $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

f) $\frac{14}{x+3} - 1 = \frac{5}{x+1}$

g) $\frac{1}{x} - \frac{1}{x-2} = 3$

h) $\sqrt{2x+9} + x = 13$

i) $\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$

j) $\left(\frac{x}{x+2}\right)^2 - 7\left(\frac{x}{x+2}\right) + 12 = 0$

k) $2x^4 - 5x^2 + 3 = 0$

l) $\sqrt{\frac{x}{1-x}} + \sqrt{\frac{1-x}{x}} = 2\frac{1}{6}$

m) $x^2 + \left(\frac{a}{a+b} + \frac{a+b}{a}\right)x + 1 = 0$

28. Solve by factorisation method: $9x^2 - 6b^2x - (a^4 - b^4) = 0$

29. **Solve by quadratic formula method:**

a) $a^2b^2x^2 - (4b^2 - 3a^4)x - \quad 12a^2b^2 = 0$
b) $9x^2 - 9(a+b)x + 2a^2 + 5ab + \quad 2b^2 = 0$
c) $abx^2 + (b^2 - 4ac)x - bc = 0$
d) $12abx^2 - (9a^2 - 8b^2)x - 6ab = 0$

30. **Solve by completing the square method:**

a) $2x^2 - 5x + 3 = 0$
b) $4x^2 + 3x + 5 = 0$
c) $4x^2 + 4\sqrt{3} + 3 = 0$
d) $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

31. **Solve:**

a) $\frac{a}{x-b} + \frac{b}{x-a} = 2$

b) $\frac{a}{ax-1} + \frac{b}{bx-1} = a + b$

c) $2^{2x} - 3.2^{x+2} + 32 = 0$

d) $4^{x+1} + 4^{1-x} = 10$

e) $(x+1)(x+2)(x+3)(x+4) = 120$

f) $9\left(x^2 + \frac{1}{x^2}\right) - 9\left(x + \frac{1}{x}\right) - 52 = 0$

g) $\left(x - \frac{1}{x}\right)^2 + 8\left(x + \frac{1}{x}\right) = 29$

h) $\left(x^2 + \frac{1}{x^2}\right) - 3\left(x - \frac{1}{x}\right) - 2 = 0$

32. **a)** If -5 is a root of the quadratic equation $2x^2 + px - 15 = 0$ and the quadratic equation $p(x^2 + x) + k = 0$ has equal roots. Find the value of p and k.
b) If 3 is a root of the quadratic equation $x^2 - x + p = 0$ and the roots of the equation $x^2 + p(2x + p + 2) + q = 0$ are equal. Find the value of p and q.

33. If the quadratic equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots. Prove that $c^2 = a^2(1 + m^2)$.

34. 35. If the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal.
Prove that $2b = a + c$.

36. Determine the positive value of p for which the equations $x^2 + 2px + 64 = 0$ and $x^2 - 8x + 2p = 0$ will both have real roots.

37. Find the value of k for each of the following quadratic equations, so that they have equal roots.

a) $(k - 12)x^2 + 2(k - 12)x + 2 = 0$
b) $kx(x - 2) + 6 = 0$
c) $kx^2 + 1 - 2(k - 1)x + x^2 = 0$

38. If $ad \neq bc$ then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.

39. If the roots of the equation $(c^2 - ab)x^2 - 2(a^2 - bc)x + (b^2 - ac) = 0$ are real and equal, show that either $a = 0$ or $(a^3 + b^3 + c^3) = 3abc$.

40. Prove that both the roots of the equation $(x-a)(x-b)+(x-b)(x-c)+(x-c)(x-a)=0$ are real but they are equal only when $a=b=c$.

POLYNOMIALS

41. Find the zeroes of the following quadratic polynomial and verify the relationship between zeroes and the coefficients.
(i) $6x^2-3-7x$ (ii) $4u^2+8u$ (iii) t^2-15

42. Obtain all the zeroes of $3x^4+6x^3-2x^2-10x-5$, if two of its zeroes are $\sqrt{\frac{5}{3}}$ and $-\sqrt{\frac{5}{3}}$.

43. On dividing x^3-3x^2+x+2 by a polynomial g (x), the quotient and remainder were $x-2$ and $-2x+4$ respectively. Find g(x)

44. Construct a quadratic polynomial whose zeros are 2 and -3.

45. Divide $3x^2-x^3-3x+5$ by $-1-x^2$.

46. If two zeros of the polynomial $x^4-6x^3-26x^2+138x-35$ are $2\pm\sqrt{3}$ find other zeros.

47. If α,β be the zeros of the quadratic polynomial x^2+8x+k such that the sum of the squares of the zeros is 40 find the value of k.

48. If α,β the zeros of quadratic polynomial x^2+6x+2 then find the value of
(i) $\alpha^{-1}+\beta^{-1}$ (ii) $\alpha-\beta$ (iii) $\alpha^3\beta^4+\alpha^4\beta^3$ (iv) $\alpha^3+\beta^3$

49. If one zero of the quadratic polynomial $(a^2+9)x^2+13x+6a$ is a reciprocal of the other find the value of a.

50. If $x=\frac{2}{3}$ and $x=-3$ are the root of the polynomial ax^2+7x+b then find the value of a & b.

51. If the polynomial $(x^4+2x^3+8x^2+12x+18)$ is divided by another polynomial (x^2+5), the remainder comes out to be $(px+q)$ find the value of p & q.

52. Find all the zeros of $(x^4+x^3-23x^2-3x+60)$ if it is given that two of its zeros are $\sqrt{3}$ and $-\sqrt{3}$.

53. If α and β are the zeros of the polynomial f(x) =x^2-5x+k such that $\alpha-\beta=1$, find the value of k.

54. If one zero of the polynomial (k-1)x^2+kx+1 is -4 then find the value of k.

55. If α,β be the zeros of the quadratic polynomial $2x^2+5x+k$ such that $\alpha^2+\beta^2+\alpha\beta=\frac{21}{4}$ then find k.

56. **a)** If α,β be the zeros of quadratic polynomial x^2+3x-2 then construct a quadratic polynomial whose zeros are $\frac{\alpha}{\beta}$ and $\frac{\beta}{\alpha}$.
b) If α and β are the zeroes of the quadratic polynomial x^2-2x+3, find a polynomial whose roots are $\frac{\alpha-1}{\alpha+1}$, $\frac{\beta-1}{\beta+1}$.
c) If α,β are the zeros of the $2x^2-5x+7$, then find a polynomial whose zeros are $2\alpha+3\beta, 3\alpha+2\beta$.

57. If 1 and -2 are two zeros of the polynomial $(x^3+4x^2-7x+10$ find its third zero.

58. It is given that -1 is one of the zeros of the polynomial $x^3+2x^2-11x-12$. Find all the zeros of the given polynomial.

59. What should be added to the polynomial $3x^3+x^2+2x+5$ so that it may the exactly divisible by $1+2x+x^2$

60. If one zero of the quadratic polynomial $2x^2+ax-6$ is 2, find the value of a. Also , find other zero.

61. α,β are zeros of the polynomial x^2-6x+a find the value of a , if $3\alpha+2\beta=20$.

62. If one zero of the polynomial (k+1)x^2-5x+5 is multiplicative inverse of the other, then find the zeros of $kn^2-3kx+9$, where k is constant.

63. Given that $x-\sqrt{5}$ is factor of the polynomial $x^3-3\sqrt{5}x^2-5x+15\sqrt{5}$, find all the zeros of the polynomial .

64. Find the values of a and b so that $x^4+x^3+8x^2+ax-b$ is divisible by x^2+1.

65. If α, β are the zeros of the polynomial p(x)=$x^2-p(x+1)-C$ such that $(\alpha+1)\ (\beta+1)=0$ what is the value of C?

66. If the zeros of the polynomial x^2+px+q are double in value to the zeros of $2x^2-5x-3$, find the value of p & q.

67. From a quadratic polynomial , one of whose zero is 2+$\sqrt{5}$ and the sum of zeros is 4.

68. m, n are zeros of ax^2-5x+c. Find the value of a and c if $m+n=m.n$ =10

69. Find k, if the sum of the zeros of the polynomial $x^2-(k+6)x+2(2k-1)$ is half the product of zeros.

70. Determine is 3 a zero of the polynomial p(x)=$\sqrt{x^2-4x+3}$+$\sqrt{x^2-9}-\sqrt{4x^2-14x+16}$.

71. If the polynomial $6x^4+8x^3-5x^2+ax+b$ is exactly divisible by the polynomial $2x^2-5$, then find the values of a and b.

72. If one zero of the polynomial $3x^2-8x+(2k+1)$ is seven times the other, find the value of k.

73. If the zeros of ax^2-bx+c be in the ratio 4:5, show that $20b^2=81ac$.

74. If one zero of the polynomial x^3-8x+k exceeds the other by 2, find the value of k.

75. If the squared difference of the zeroes of the quadratic polynomial $f(x)=x^2+px+45$ is equal to 144, find the value of p.

LINEAR EQUATION

76. Solve the following pair of linear equations:

i) $\sqrt{2}\,x+\sqrt{3}\,y$ = 0
$\sqrt{3}\,x+\sqrt{8}\,y$ = 0

ii) $\frac{3x}{2}-\frac{5y}{3}$ = – 2
$\frac{x}{3}-\frac{y}{2}=\frac{13}{6}$

77. Solve $2x\ +\ 3y$ = 11 and $2x$ – $4y$ = – 24 and hence find the value of 'm' for which $y\ =\ mn$ + 3.

78. Solve: $\frac{x}{2}+\frac{2y}{3}=-1$ and $x-\frac{y}{3}=3$

79. For which values of a and b does the following pair of linear equations have an infinite number of solutions?

80. $2x+3y$ = 7
$(a-b)x+(a+b)y=3a+b-2$

81. For which value of k will the following pair of linear equations have no solution?

82. $3x+y$ = 1
$(2k-1)x+(k-1)y=2k+1$

83. Solve the following pair of linear equations:

i) $\frac{1}{2x}+\frac{9}{3y}=2$
$\frac{1}{3x}+\frac{1}{2y}=\frac{13}{6}$

ii) $\frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2$

$\frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1$

iii) $\frac{4}{x} + 3y = 14$

$\frac{3}{x} + 4y = 23$

iv) $\frac{5}{x-1} + \frac{1}{y-2} = 2$

$\frac{6}{x-1} - \frac{3}{y-2} = 1$

v) $\frac{7x-2y}{xy} = 5$

$\frac{8x+7y}{xy} = 15$

vi) $\frac{10}{x+y} + \frac{2}{x-y} = 4$

$\frac{15}{x+y} - \frac{5}{x-y} = -2$

vii) $\frac{1}{3x+y} + \frac{1}{3x-y} = \frac{3}{4}$

$\frac{1}{2(3x+y)} - \frac{1}{2(3x-y)} = \frac{-1}{8}$

84. Solve the following pair of linear equations:

i) $px + qy = p - q$

$qx - py = p + q$

ii) $(a-b)x + (a+b)y = a^2 - 2ab - b^2$

$(a+b)(x+y) = a^2 + b^2$

iii) $\frac{x}{a} - \frac{y}{b} = 0$

$ax + by = a^2 + b^2$

iv) $152x - 378y = -74$

$-378x + 152y = -604$

TRIGONOMETRY

85. If 5 cos θ = 3, find the value of $\frac{5\operatorname{cosec}\theta - 4\tan\theta}{\sec\theta + \cot\theta}$

86. If 8 cot θ = 15, evaluate $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

87. If cosec θ =2, evaluate $\frac{\cot\theta + \sin\theta}{1+\cos\theta}$

88. If tan θ = $\frac{1}{\sqrt{7}}$, evaluate $\frac{\operatorname{cosec}^2\theta - \sec^2\theta}{\operatorname{cosec}^2\theta + \sec^2\theta}$

89. If tan θ = $\frac{20}{21}$, evaluate $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta}$

90. If sec θ = $\frac{17}{8}$, verify that $\frac{3-4\sin^2\theta}{4\cos^2-3} = \frac{3-\tan^2\theta}{1-3\tan^2\theta}$

91. If $\tan\theta = \frac{a}{b}$, evaluate $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta}$

92. Evaluate $\frac{\tan 60^0 + 4\sin^2 45 + 3\sec^2 30^0 + 5\cos^2 90^0}{\text{cosec} 30^0 + \sec 60^0 - \cot 30}$

93. Evaluate $\sin^2 30^0 . \cos^2 45^0 + 4\tan^2 30^0 + \frac{1}{2}\sin^2 90^0 + \frac{1}{8}\cot^2 60^0$

94. If A = 60° and B = 30° verify that $\tan(a - b) = \frac{\tan A - \tan B}{1 + \tan A \times \tan B}$

95. Using sin (A – B) = $\sin A \times \cos B + \cos A \times \sin B$. find the value of sin 15.

96. Using cos A = $\sqrt{\frac{1+\cos 2A}{2}}$, find cos 300 given that cos 600 = $\frac{1}{2}$

97. Find the value of sin60°, cos30° geometrically

98. Find the value of sin 45° geometrically.

99. Evaluate: $\frac{\cos^2 20^0 + \cos^2 70^0}{\sec^2 50^0 - \cot^2 40^2} + 2\text{cosc}^2 58^0 - 2\cot 58^0 . \tan 32^0 - 4\tan 13^0 . \tan 37^0 . \tan 45^0 . \tan 58^0 . \tan 77^0$

100. Evaluate $\tan 1^0 . \tan 2^0 . \tan 3^0 \ldots\ldots\ldots\ldots . \tan 89^0$

101. Find the value of $\cos 1^0 . \cos 2^0 . \cos 3^0 \ldots\ldots\ldots\ldots . \cos 90^0$

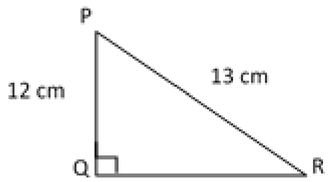
102. If $\sqrt{3}\tan 2\theta - 3 = 0$, find θ

103. If tan θ = $\sqrt{2} - 1$, find the value of sin θ.cos θ

104. In ΔOPQ right angled at P, OP = 7 cm and OQ – PQ = 1 cm. determine the values of sin θ and cos θ

105. In figure, find tan P – cot R

106.



If 3 cot A = 4, check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not

107. In ΔABC right angled at B, if tan A = $\frac{1}{\sqrt{3}}$, find the value of

i) sin A cos C + cos A sind C ii) cos A cos C – sin A sin C

108. In ΔPQR right angled at Q, PR + QR = 25 cm & PQ = 5 cm. determine the value of sin P, cos P and tan P.

109. In ΔABC, right angled at B, AB = 5 cm and ∠ACB = 30°. Determine the length of the sides BC and AC.

110. In ΔPQR right angled at Q, PQ = 3 cm and PR = 6 cm. determine ∠QPR and ∠PRQ.

111. If sin (A – B) = $\frac{1}{2}$, cos (A + B) = $\frac{1}{2}$, 00 ∠A + B ≤ 900, A > B, find A and B.

112. Evaluate: i) $\frac{\cos 45^0}{\sec 30^0 + \text{cosec} 30^0}$ ii) $\frac{\sin 30^0 + \tan 45^0 - \text{cosec} 60^0}{\sec 30^0 + \cos 60^0 + \cot 45^0}$

113. Evaluate: $\text{cosec}\, 31^0 - \sec 59^0$

114. If $\tan A = \cot B$, prove that A + B = 90°

115. If sec 4A = Cosec (A-20°), where 4A is an acute angle, find the value of A.

116. If A, B and C are interior angle of a ΔABC, then show that $\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$

117. Express sin 67° + cos 75° in form of T- ratio of angles between 0° and 45°.

118. Evaluate: $4(\sin^4\theta + \cos^4 60^0) - 3(\cos^2 45^0 - \sin^2 90^0)$

119. If sin(A + B) = 1 and tan (A – B) = $\frac{1}{\sqrt{3}}$ find the value of sec A – cosec B

120. If $15\tan^2\theta + 4\sec^2\theta = 23$, then find the value of $(\sec\theta + \mathrm{cosec}\theta)^2 - \sin^2\theta$

121. In ΔABC, ∠A = 900, AB = $\sqrt{x}$ and BC = $\sqrt{x+5}$. evaluate : sin C.cos C. tan C + $\cos^2 C.\sin A$

122. Solve for θ, $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4$

123. Evaluate: cosec(65°+θ) – sec(26° – θ) – tan(55° – θ) + cot(35° + θ) = 0

124. Evaluate: $\frac{\tan\theta.\cot(90^0-\theta)+\sec\theta\ \mathrm{cosec}(90^0-\theta)+\sin^2 35^0+\sin^2 55^0}{\tan 10^0 \tan 20^0 \tan 30^0 \tan 70^0 \tan 80^0}$

125. Evaluate: $\frac{3\tan 25^0 \tan 40^0 \tan 80^0 \tan 65^0 - \frac{1}{2}\tan^2 60^0}{4(\cos^2 29^0 + \cos^2 61^0}$

126. Evaluate: $\frac{\cos^2 20^0+\cos^2 70^0}{\sin^2 20^0+\sin^2 70^0} + \sin^2 64^0 + \cos 64^0 \sin 26^0$

127. Evaluate: $\sec^2 10^0 - \cot^2 80^0 + \frac{\sin 15^0 \cos 75^0 + \cos 15^0 \sin 75^0}{\cos\theta\sin(90^0-\theta+\sin\theta\cos(90^0-\theta)}$

128. Evaluate: $\frac{\cos^2 20^0+\cos^2 70^0}{\sec^2 50^0-\cot^2 40^0} + 2\mathrm{cosec}^2 58^0 - 2\cot 58^0 \tan 32^0$

129. Evaluate: $\frac{\mathrm{cosec}^{2(}90^{0-}\theta)-\tan^2\theta}{4(\cos^2 48^0+\cos^2 42^0} - \frac{2\tan^2 30^0 \sec^2 52^0 \sin^2 38^0}{(\mathrm{cosec}^2 70^0 - \tan^2 20^0}$

130. Evaluate: $\frac{\sec 39^0}{\mathrm{cosec} 51^0} + \frac{2}{\sqrt{3}}.\tan 17^0 \tan 38^0 \tan 60^0 \tan 52^0 \tan 75^0 - 3(\sin^2 31^0 + \sin^2 59^0$

TRIGONOMETRIC IDENTITIES

131. $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{\mathrm{cosec} A - 1}{\mathrm{cosec} A + 1}$

132. $(\mathrm{cosec}\theta - \cot\theta)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

133. $\frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A} = 2\sec A$

134. $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

135. $\frac{1+\sec A}{\sec A} = \frac{\sin^2 A}{1-\cos\theta}$

136. $(\sin A + \mathrm{cosec} A)^2 + (\cos A + \sec A)^2 = 7 + \tan^2 A + \cot^2 A$

137. $(\mathrm{cosec}\ A - \sin A)(\sec A - \cos A) = \frac{1}{\tan A + \cot A}$

138. $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right) = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$

139. $(1+\tan\theta+\sec\theta)(1+\cot\theta-\mathrm{cosec}\ \theta) = 2$

140. $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1} = \frac{1}{\sec\theta-\tan\theta}$

141. $\frac{\cos A-\sin A+1}{\cos A+\sin A-1} = \mathrm{cosec} A + \cot A$

142. $\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\ \mathrm{cosec}\theta$

143. $\frac{\cos\theta}{(1-\tan\theta)} + \frac{\sin\theta}{(1-\cot\theta)} = (\cos\theta + \sin\theta)$

144. $\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta \mathrm{cosec}^2\theta$

145. $(\sec^4\theta - \sec^2\theta) = (\tan^2\theta + \tan^4\theta)$

146. $\left(1+\frac{1}{\tan^2 A}\right)\left(1+\frac{1}{\cot^2 A}\right) = \frac{1}{(\sin^2 A - \sin^4 A)}$

147. $\frac{1}{(\operatorname{cosec}\theta - \cot\theta)} - \frac{1}{\sin\theta} = \frac{1}{\sin\theta} - \frac{1}{(\operatorname{cosec}\theta + \cot\theta)}$

148. $\frac{\sec\theta + \tan\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\cos\theta}{(1-\sin\theta)}$

149. $\sqrt{\sec^2\theta + \operatorname{cosec}^2\theta} = \tan\theta + \cot\theta$

150. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$

151. $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta} = 2$

152. $\frac{\sin\theta}{(\cot\theta + \operatorname{cosec}\theta)} = 2 + \frac{\sin\theta}{(\cot\theta - \operatorname{cosec}\theta)}$

153. $\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} + \frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} = \frac{2}{(2\sin^2\theta - 1)}$

154. $\frac{1+\cos\theta - \sin^2\theta}{\sin\theta\ (1+\cos\theta)} = \cot\theta$

155. $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3 - \cos\theta} = \tan\theta$

156. $\frac{\operatorname{cosec}\theta + \cot\theta}{\operatorname{cosec}\theta - \cot\theta} = (\operatorname{cosec}\theta + \cot\theta)^2 = 1 + 2\cot^2\theta + 2\operatorname{cosec}\theta\cot\theta$

157. $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{2}{(\sin^2\theta - \cos^2\theta)} = \frac{2}{(2\sin^2\theta - 1)}$

158. $\sin\theta(1+\tan\theta) + \cos\theta(1+\cot\theta) = (\sec\theta + \operatorname{cosec}\theta)$

159. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

160. $2\sec^2\theta - \sec^4\theta - 2\operatorname{cosec}^2\theta + \operatorname{cosec}^4\theta = \cot^4\theta - \tan^4\theta$

161. $(1-\sin\theta + \cos\theta)^2 = 2(1+\cos\theta)(1-\sin\theta)$

162. $(1+\cot A + \tan A)(\sin A - \cos A) = \frac{\sec A}{\operatorname{cosec}^2 A} - \frac{\operatorname{cosec} A}{\sec^2 A} = \sin A\tan A - \cot A\cos A$

163. $\frac{(1+\sin\theta)^2 + (1-\sin\theta)^2}{\cos^2\theta} = 2\left(\frac{1+\sin^2\theta}{1-\sin^2\theta}\right)$

164. $(\sec\theta - \operatorname{cosec}\theta)(1+\tan\theta+\cot\theta) = (\sec\theta\tan\theta - \operatorname{cosec}\theta\cot\theta)$

165. $(\sin\theta + \sec\theta)^2 + (\cos\theta + \operatorname{cosec}\theta)^2 = (1+\sec\theta\operatorname{cosec}\theta)^2$

166. $\left(\frac{\cos A}{1-\sin A} + \frac{1+\sin A}{\cos A}\right) - \left(\frac{\cos A}{1-\sin A} - \frac{1-\sin A}{\cos A}\right) = 4\tan A.\sec A$

ELIMINATION OF TRIGONOMETRIC RATIOS

167. If $\tan\theta + \sin\theta = m$ and $\tan\theta - \sin\theta = n$, prove that $(m^2 - n^2) = 4\sqrt{mn}$

168. If $\sec\theta + \tan\theta = m$, show that $\frac{m^2-1}{m^2+1} = \sin\theta$

169. If $\sin\theta + \cos\theta = m$ and $\sec\theta + cosec\theta = n$, prove that $n(m^2-1) = 2m$.

170. If $\sin\theta + \sin^2\theta = 1$, prove that $\cos^2\theta + \cos^4\theta = 1$.

171. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\ \cos\theta$ and $xsin\ \theta = ycos\ \theta$, prove that $x^2 + y^2 = 1$.

172. If $x = a\ \sec\theta\cos\phi, y = b\sec\theta\sin\phi$ and $z = c\tan\theta$ then prove that $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = \left(1 + \frac{z^2}{c^2}\right)$

173. If $x = r\sin\alpha\cos\beta$, $y = r\sin\alpha\sin\beta$ and $z = r\cos\alpha$, prove that $x^2 + y^2 + z^2 = r^2$.

174. If $cosec\theta - \sin\theta = m$ and $\sec\theta - \cos\theta = n$, prove that $(m^2 n)^{\frac{2}{3}} + (m\,n^2)^{\frac{2}{3}} = 1$

175. If $a\cos\theta - b\sin\theta = c$, prove that $(\ a\sin\theta + b\cos\theta\ ) = \pm\sqrt{a^2+b^2-c^2}$

176. If $x = a\sec\theta + b\tan\theta$ and $y = a\tan\theta + b\sec\theta$, prove that $(x^2 - y^2) = (a^2 - b^2)$

177. If $\left(\frac{x}{a}\sin\theta - \frac{y}{b}\cos\theta\right) = 1$ and $\left(\frac{x}{a}\cos\theta + \frac{y}{b}\sin\theta\right) = 1$, prove that $\left(\frac{x^2}{a^2} + \frac{y^2}{b^2}\right) = 2$

178. If $\tan A = n\tan B$ and $\sin A = m\sin B$, prove that $\cos^2 A = \frac{(m^2-1)}{(n^2+1)}$

179. If $\frac{\cos\alpha}{\cos\beta} = m$ and $\frac{\cos\alpha}{\sin\beta} = n$, show that $(m^2 + n^2)\cos^2\beta = n^2$

180. If $\sec\theta = x + \frac{1}{4x}$, prove that $\sec\theta + \tan\theta = 2x$ or $\frac{1}{2x}$

181. If $5x = \sec\theta$ and $\frac{5}{x} = \tan\theta$. Find the value of $5\left(x^2 - \frac{1}{x^2}\right)$

182. If $\sec\theta - \tan\theta = x$, show that $\sec\theta = \frac{1}{2}\left(x + \frac{1}{x}\right)$ and $\tan\theta = \frac{1}{2}\left(\frac{1}{x} - x\right)$

183. (a) Express the ratios of $\cos A, \tan A$ and $\sec A$ in terms of $\sin A$. (b) Express the T – ratios $\sin A$, $\sec A$ and $\tan A$ in terms of $\cot A$.

184. If $\cos A - \sin A = m$ and $\cos A + \sin A = n$, show that : $\frac{m^2-n^2}{m^2+n^2} = -2\sin A\cos A = \frac{2}{\tan A + \cot A}$

185. If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$ prove that $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

HEIGHTS AND DISTANCES

186. A tower stands vertically on the ground. From a point on the ground. Which is 15 m away from the foot of the tower, the angle of elevation of the top of the tower is found to 60^0. Find the height of the tower.

187. From a point p on the ground the angle of elevation of the top of a 10 m + all building is 30^0. A flag is hoisted at the top of a building and the angle of elevation of the top of the flagstaff from P is 45^0, find the length of the flag staff and the distance of the building from the point P ($\sqrt{3}$ = 1.723)

188. The shadow of a tower standing on a level ground is found to be 40 m longer when the seen altitude is 30^0 than when it is 60^0, find the height of the tower.

189. The angles of depression of the top and the bottom of an 8 m tall building from the top of a multi-storeyed building are 30^0 and 45^0 respectively. Find the height of the multi-storeyed building and the distance between the two buildings.

190. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle of 30^0 with it. The distance between the foot of the tree to the point when the top touches the ground is 8 m, find the height of the tree.

191. A 1.5 m tall boy is standing at some distance from a 30 m tall building. The angle of elevation from his eyes to the top of the building increases from 30^0 to 60^0 as he walks towards the building find the distance he walks towards the building.

192. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45^0 and 60^0 respectively; find the height of the tower.

193. The angle of elevation of the top of a building from the foot of the tower is 30^0 and the angle of elevation of the top of the tower from the foot of the building is 60. If the tower is 50 m high, find the height of the building.

194. Two poles of equal heights are standing opposite each other on either side of the road. Which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60^0 and 30^0 respectively; find the height of the poles and the distances of the point from the poles.

195. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30^0 and 45^0. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

196. A straight highway leads to the foot of a tower. A man standing at the top of the tower observes a car at an angle of depression of 30^0, which is approaching the foot of the tower with a uniform speed. Six seconds later, the angle of depression of the car is found to be 60^0, find the time taken by the car to reach the foot of the tower from this point.

197. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary prove that the height of the tower is 6 m.

CONSTRUCTIONS

198. Draw a line segment of length 7.6 cm and divide it in the ratio 5 : 8. Measure the two parts.

199. Construct a triangle of sides 4 cm, 5 cm and 6 cm and then a triangle similar to it whose sides are $\frac{2}{3}$ of the corresponding sides of the first triangle.
200. Draw a triangle ABC with side BC = 6 cm, AB = 5 cm and $\angle ABC = 60^0$. Then construct a triangle whose sides are $\frac{3}{4}$ of the corresponding sides of the triangle ABC.
201. Construct a triangle with sides 5 cm, 6 cm and 7 cm and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.
202. Construct an Isosceles triangle whose base is 8 cm and altitude 4 cm and then another triangle whose sides are $1\frac{1}{2}$ times the corresponding sides of the Isosceles triangle.
203. Draw a circle of radius 6 cm from a point 10 cm away from its centre, construct the pair of tangents to the circle and measure their lengths.
204. Construct a tangent to a circle of radius 4 cm from a point on the concentric circle of radius 6 m and measure its lengths. Also verify the measurement by actual calculation.
205. Draw a pair of tangents to a circle of radius 5 cm which are inclined to each other at an angle of 60^0.
206. Draw a line segment AB of length 8 cm. taking A as centre, draw a circle of radius 4 cm and taking B as centre, draw another circle of radius 3 cm. construct tangents to each circle from the centre of the other circle.
207. Let ABC be aright triangle in which AB = 6 cm, BC = 8 cm and $\angle B = 90^0$, BD is the perpendicular from B on AC. The circle through B, C, D is drawn construct the tangents from A to this circle.
208. Draw a circle of radius 5 cm. take a point P on it, without using the centre of the circle construct a triangle at the point P.
209. Draw a circle of radius 3.5 cm draw two tangents to the circle which are perpendicular to each other.
210. Draw a circle of radius 3 cm. take two points P and Q on one of its extended diameter each at a distance of 7 cm from its centre. Draw tangents to the circle from these two points P and Q.

VOLUME AND SURFACE AREAS OF SOLIDS

211. A sphere and a cube have equal surface area. Show that the ratio of the volume of sphere to that of the cube is $\sqrt{6}:\sqrt{\pi}$
212. A vessel is in the form of a hemispherical bowl mounted by a hollow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. find i) the capacity of the vessel ii) the inner surface area of the vessel.
213. A solid wooden toy is in the form of a hemisphere surrounded by a cone of same radius. The radius of the hemisphere is 3.5 cm and the total wood used in the making of toy is $166\frac{5}{6}cm^3$. Find the height of the toy. Also find the cost of painting the hemispherical part of the toy at the rate of `10 per cm^2.
214. A solid cylinder of diameter 12 cm and height 15 cm is melted and recast into 12 toys in the shape of a right circular cone mounted on a hemisphere. Find the radius of the hemisphere and total height of the toy, if the length of the cone is 3 times the radius.
215. The largest possible sphere is curved out from a solid wooden cube of side 7 cm. find
216. i) the volume of the sphere ii) the percentage of wood wasted in the process.
217. The volume of a hemisphere is $2425\frac{1}{2}cm^3$, find its curved surface area.
218. The radius and height of a solid right-circular cone are in the ratio of 5 : 12. If its volume is 314 cm^3, find its total surface area. [take $\pi = 3.14$]
219. A metallic sphere of radius 10.5 cm is melted and then recast into smaller cones, each of radius 3.5 cm and height 3 cm. how many cones are obtained?
220. The internal and external radii of a hollow sphere are 3 cm and 5 cm respectively. The sphere is melted to form a solid cylinder of height $2\frac{2}{3}$ cm. find the diameter and the curved surface area of the cylinder.

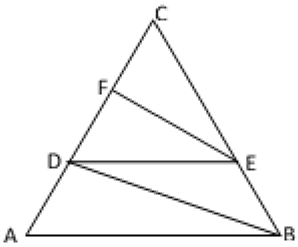
221. A hollow sphere of internal and external diameters 4 cm and 8 cm is melted to form a cone of base diameter 8 cm. find the height and the slant height of the cone.
222. 504 cones, each of diameter 3.5 cm and height 3 cm, are melted and recast into a metallic sphere. Find the diameter of the sphere and hence find its surface area.
223. A hemispherical bowl of internal radius 9 cm is full of liquid. The liquid is to be filled into cylindrical shaped small bottles, each of diameter 3 cm and height 4 cm. how many bottles are needed to empty the bowl.
224. A hemispherical bowl of internal diameter 36 cm contains liquid. This liquid is filled into 72 cylindrical bottles of diameter 6 cm. find the height of each bottle if 10% liquid is wasted in this transfer.
225. Water flows through a circular pipe whose internal diameter is 2 cm, at the rate of 0.7 m per second into a cylindrical tank, the radius of whose base is 40 cm. by how much will the level of water rise in the tank in half an hour?
226. Water is flowing at the rate of 15 km per hour through a pipe of diameter 14 cm into a rectangular tank which is 50 m long and 44 m wide, find the time in which the level of water in the tank will rise by 21 cm.
227. Water is flowing at the rate of 2.52 km/hr through a cylindrical pipe into a cylindrical tank, the radius of whose base is 40 cm. if the increase in the level of water in the tank, in half an hour is 3.15 m, find the internal diameter of the pipe.
228. A spherical ball of radius 3 cm is melted and recast into three spherical balls. The radii of two of these balls are 1.5 cm and 2 cm. find the radius of the third ball.
229. Water running in an cylindrical pipe of inner diameter 7 cm, is collected in a container at the rate of 192.5 litres per minute. Find the rate of flow of water in the pipe in km/hr.
230. 150 spherical marbles, each of diameter 14 cm, are chopped in a cylindrical vessel of diameter 7 cm containing some water, which are completely immersed in water. Find the rise in the level of water in the vessel.
231. A bucket is in the form of a frustum of a cone with a capacity of 12308.8 cm^3 of water. The radii of the top and bottom circular ends are 20 cm and 12 cm respectively. Find the height of the bucket and the area of the metal sheet used in its making. [use π=3.14]
232. The height of a cone is 10 cm. the cone is divided into two parts using a plane parallel to the base at the middles of its height. Find the ratio of the volumes of the two parts.
233. The height of a cone is 30 cm. a small cone is cut off at the top by a plane parallel to the base. If its volume be 1/27 of the volume of the given cone, at what height above the base is the section made?
234. A hollow cone is cut by a plane parallel to the base and the upper portion is removed. If the curved surface of the remainder is 8/9 of the curved surface of the whole cone, find the ratio of the line segments into which the altitude of the cone is divided by the plane.
235. A right circular cone is divided into three parts by trisecting its height by two planes drawn parallel to the base. Show that the volumes of the three portions starting from the top are in the ratio 1 : 7 : 19.
236. The ratio between the radius of the base and the height of a cylinder is 2 : 3. If the volume of the cylinder is 12936 cm^3, find the radius of the base of the cylinder.
237. Three cubes of a metal whose edges are in the ratio 3 : 4 : 5 are melted and converted into a single cube whose diagonal is $12\sqrt{3}$ cm. find the edges of the three cubes.

TRIANGLE

238. Prove that if a line is drawn parallel to one side of a triangle to intersect the other two side are divided in the same ratio.
239. Prove that the ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.
240. Prove that in a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.

241. Prove that, in a right triangle, if the square on one side is equal to the sum of the squares on the other two sides, the angle opposite to the first side is a right angle.

242. In the given figure, AB || DE and BD || EF. Prove that $DC^2 = CF \times AC$.

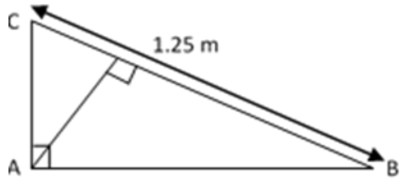


243. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AD}{OC} = \frac{BO}{OD}$ show that ABCD is trapezium.

244. Two right triangles ABC and DBC are drawn on the same hypotenuse BC and on the same side of BC. If AC and BD intersect at P, prove that AP × PC = BP × PD.

245. Through the midpoint M of the side CD of a parallelogram ABCD, the line BM is drawn, intersecting AC in L and AD produced in E. prove that EL = 2BL.

246. In the given figure, $\angle CAB = 90^0$ and $AD \perp BC$. Show that $\Delta BDA \sim \Delta BAC$. If AC = 75 cm, AB = 1 m and BC = 1.25 m, find AD.



COORDINATE GEOMETRY

247. Show that the points (1, 7), (4, 2), (-1, -1) and (-4, 4) are the vertices of a square and find its area.

248. Find a point on the y –axis which is equidistant from the points A(6, 5) and B(-4, 3).

249. Find the point on the $x -$ axis which is equidistant from (2, -5) and (-2, 9).

250. Find the value of y for which the distance between the points P(2, - 3) and Q(10, y) is 10 units.

251. If Q(0,1) is equidistant from P(5, -3) and R(x, 6), find the value of x. Also find the distances QR and PR.

252. Find a relation between x and y such that the point (x, y) is equidistant from the point (3, 6) and (-3, 4).

253. Find the coordinates of the points of trisection (i.e. points dividing in three equal parts) of the line segment joining the points A(2, -2) and B(-7, 4).

254. Find the ratio in which the line segment joining A(1, -5) and B(-4, 5) is divided by the $x -$ axis. Also find the coordinates of the point of division.

255. If (1, 2), (4, y), (x, 6) and (3, 5) are the vertices of a parallelogram taken in order, find x and y.

256. Find the coordinates of a point A, where AB is the diameter of a circle whose centre is (2, -3) and B is (1, 4).

257. If A and B are (-2, -2) and (2, -4), respectively find the coordinates of P such that AP = $\frac{3}{7}$ AB and P lies on the line segment AB.

258. Find the coordinates of the points which divide the line segment joining A(-2, 2) and B(2, 8) into four equal parts.

259. Find the area of a rhombus if its vertices are (3, 0), (4, 5), (-1, 4) and (-2, -1) taken in order.

260. If A(-5, 7), B(-4, -5), C(-1, -6) and D(4, 5) are the vertices of a quadrilateral, find the area of the quadrilateral ABCD.

261. Find the value of 'k' if the points A(8, 1), (k, - 4) and (2, -5) are collinear.

262. Find the area of the triangle formed by joining the mid – points of the sides of the triangle whose vertices are (0, -1), (2, 1) and (0, 3). Find the ratio of this area to the area of the given triangle.

263. A median of a triangle divides it into two triangles of equal areas. Verify this result for ΔABC whose vertices are A(4, -6), B(3, -2) and C(5, 2).

264. Determine the ratio in which the line $2x + y - 4 = 0$ divides the line segment joining the points A(2, -2) and B(3, 7).
265. Find a relation between x and y if the points (x, y), (1, 2) and (7, 0) are collinear.
266. Find the centre of a circle passing through the points (6, -6), (3, -7) and (3, 3).
267. Find those points on x-axis, each of which is at a distance of 5 units from point A(5, -3).
268. Find the coordinates of point equidistant from three given points A(5, 1), B(-3, -7) and C(7, - 1).
269. Points A(1, y) and B(5, 7) lie on a circle with centre O(2, 3y). Find the value of y. Hence, find the radius of the circle.
270. The points A(4, 7), B(p, 3) and C(7, 3) are the vertices of a right triangle, right angled at B. Find the value of p.
271. Show that the points (a, a), $(-a, -a)$ and $(-\sqrt{3}a, \sqrt{3}a)$ are the vertices of an equilateral triangle. Find the area.
272. Prove that the points A(3, 0), B(1, -3) and C(4, 1) are the vertices of an isosceles right angled triangle. Find its area.
273. Show that the points (1, 1), (-1, 5), (7, 9), (9, 5) taken in that order are vertices of rectangle.
274. If A(2, -1), B(3, 4), C(-2, 3) and D(-3, -2) be four points in a plane, show that ABCD is a rhombus but not a square. Find area.
275. Using distance formula, prove that the points A(1, 1), B(-2, 7) and C(3, - 3) are collinear.
276. If the points A(0, 2) is equidistant from the points B(3, p) and C(p, 5), find the value of p. Also find the length of AB.
277. If the point P(x, y) is equidistant from the points A(5, 1) and B(- 1, 5). Prove that $3x = 2y$.
278. If the point (x, y) is equidistant from the points A$(a + b, b - a)$ and $(a - b, a + b)$, prove that $bx = ay$.

279. Find the length of the median AD and BE of ΔABC whose vertices are A(1, -3), B(5, 3) and C(3, -1).
280. The three vertices of a parallelogram ABCD taken in order are A(3, -4), B(-1, -3) and C(- 6, 2). Find the coordinates of the fourth vertex D.
281. Let D(3, -2), E(-3, 1) and F(4, -3) be the mid – points of the sides BC, CA and AB respectively of ΔABC. Then, find the coordinates of the vertices A, B and C.
282. Point A lies on the line segment PQ joining P(6, - 6) and Q(-4, -1) in such a way the $\frac{\text{PA}}{\text{PQ}} = \frac{2}{5}$. If the point A also lies on the line. $3x + k(y + 1) = 0$, find the value of k.
283. The line segment joining the points A(3, - 4) and B(1, 2) is trisected at the points P(p, -2) and P$\left(\frac{5}{3}, q\right)$. Find the values of p and q.
284. The mid - point of the line segment joining A(2*a*, 4) and B(-2, 3*b*) is C(1, 2*a*+1). Find the values of *a* and *b*.
285. If G(-2, 1) is the centroid of a ΔABC and two of its vertices are A(1, -6) and B(-5, 2), find the third vertex of the triangle.
286. Find the third vertex of a ΔABC if two of its vertex are B(-3, 1) and C(0, -2), and its centroid is at the origin.
287. The base QR of an equilateral ΔPQR lies on *x* – axis. The coordinates of the point Q are (-4, 0) and origin is the mid – point of the base. Find the coordinates of the points P and R.
288. The base BC of an equilateral ΔABC lies on *y* – axis. The coordinates of point C are (0, -3). The origin is the mid – point of the base. Find the coordinates of the point A and B. Also, find the coordinates of another point D such that ABCD is a rhombus.
289. ABCD is a rectangle formed by the points A(-1, -1), B(-1, 4), C(5, 4) and D(5, -1). If P, Q, R and S be the mid – points of AB, BC, CD and DA respectively, show that PQRS is a rhombus.
290. The mid – point P of the line segment joining the points A(-10, 4) and B(-2, 0) lies on the line segment joining the points C(-*q*, -4) and D(-4, *y*) Find the ratio in which P divides CD. Also find the value of *y*.

291. Find the value of k so that the area of the Δ with vertices (-1, 1), (-4, 2k) and (-k, -5) is 24 square units.
292. If the points A(1, -2), B(2, 3), (-3, 2) and D(-4, -3) are the vertices of a parallelogram ABCD then taking AB as the base, find the height of the parallelogram.
293. Show that the points A(a, b+c), B(b, c + a) and C(c, a + b) are collinear.
294. If the area of ΔABC with vertices A(x, y), B(1, 2) and C(2, 1) is 6 square units then prove that $x + y = 15$ or $x + y + 9 = 0$.
295. Find the value of k for which the points A(k +1, 2k), B(3k, 2k + 3) and C(5k-1, 5k) are collinear.
296. If R(x, y) is a point on the line segment joining the points P(a, b) and Q(b, a), then prove that $x + y = a + b$.
297. Prove that the points A(a, 0), B(0, b), C(1, 1) are collinear, if $\frac{1}{a} + \frac{1}{b} = 1$.
298. Points P, Q, R and S divide the line segment joining the points A(1, 2) and B(6, 7) in five equal parts. Find the coordinates of the points P, Q and R.
299. If the joining A(-1, -4), B(b, c) and C(5, -1) are collinear and 2b + c = 4, find the values of b and c.
300. The two opposite vertices of a square are (-1, 2) and (3, 2). Find the coordinates of the other two vertices,
301. Let A(4, 2), B(6, 5) and C(1, 4) be the vertices of ΔABC.
(i) The median from A meets BC at D. Find the coordinates of the point D.
(ii) Find the coordinates of the point P on AD such that AP : PD = 2 : 1.
(iii) Find the coordinates of points Q and R on medians BE and CF respectively such that BQ : QE = 2 : 1 and CR : RF = 2 : 1.
(iv) What do you observe?
302. If 'a' is the length of one of the sides of an equilateral triangle ABC, base BC lies on x-axis and vertex B is at the origin. Find the co-ordinates of the vertices of the triangle ABC
303. The line joining the points (2, 1) and (5, -8) is trisected at the points P and Q. If point P lies on the line $2x - y + k = 0$. Find the value of k.
304. If A(4, -8), B(3, 6) and C(5, -4) are the vertices of ΔABC. D is the mid – point of BC and P is a point on AD joined such that $\frac{\text{AP}}{\text{PD}} = 2$. Find the coordinates of P.
305. A(6, 1), B(8, 2) and C(9, 4) are three vertices of a parallelogram ABCD. If E is the midpoint of DC. Find the area of ΔADE.
306. Points A(-1, y) and B(5, 7) lie on a circle with centre O(2, -3y). Find the value of y. Hence, find the radius of the circle.
307. The mid – point of the line segment joining the points A(-10, 4) and B(-2, 0) lies on the line segment joining the points C(-9, -4) and D(-4, y). Find the ratio in which P divides CD. Also, find the value of y.

CIRCLES

308. Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.
309. Prove that the lengths of tangents drawn from an external point to a circle are equal.
310. Prove that the parallelogram circumscribing a circle is a rhombus.
311. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.
312. Two tangents TP and TQ are drawn to a circle with centre O from an external point T.
313. Prove that ∠PTQ = 2 ∠OPQ
314. PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T Find the length TP.
315. Prove that the intercept of a tangent between two parallel tangents to a circle subtends a right angle at the centre.

316. A triangle ABC is drawn to circumscribe a circle of radius 4 cm such that the segments BD and DC into which BC is divided by the point of contact D are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.

317. Prove that the angle between the two tangents drawn from an external point to a circle is supplementary to the angle subtended by the line – segment joining the points of contact at the centre.

318. Two concentric circles are of radii 5 cm and 3 cm. Find the length of the chord of the larger circle which touches the smaller circle.

319. Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact.

320. The incircle of ΔABC touches the sides BC, CA and AB at P, Q and R respectively.

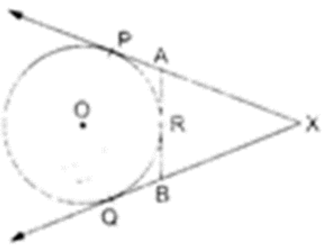
321. Prove that (AR+BP+CQ) = (AQ+BR+CP) = $\frac{1}{2}$ (perimeter of ΔABC)

322. A circle is touching the side BC of ΔABC at P and touching AB and AC produced at Q and R respectively. Prove that AQ= $\frac{1}{2}$ (perimeter of ΔABC)

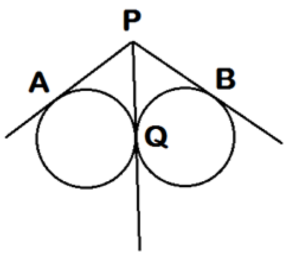
323. The sides AB, BC and CA of a triangle ABC touch a circle with centre O and radius r at P, Q and R respectively. Prove that (a) AB + CQ = AC + BQ (b) area (ΔABC) = $\frac{1}{2}$ (perimeter of ΔABC) $\times r$

324. ABC is a right – angled triangle with AB = 6 cm and AC = 8 cm. A circle with centre O has been inscribed inside the triangle. Calculate the value of r, the radius of the inscribed circle.

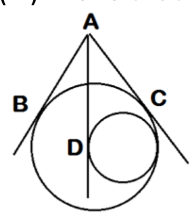
325. (i) Prove that XA + AR = XB + BR.



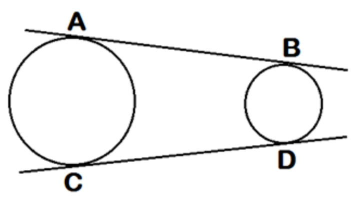
(ii) Prove that PA = PB.



(iii) Prove that AB = AD



(iv) Prove that AB = CD.



326. ABCD is a quadrilateral such that ∠D = 90°. A circle with centre O and radius r, touches the sides AB, BC, CD and DA at P, Q, R and S respectively. If BC = 40 cm, CD = 25 cm and BP = 28 cm, find r.

327. From a point P, two tangents PA and PB are drawn to a circle C(O, r). If OP = 2r, Show that ΔAPB is equilateral.

328. The incircle of an isosceles triangle ABC, with AB = AC, whose touches the sides AB, BC, CA at D, E and F respectively. Prove that E bisects BC.

329. If from an external point P of a circle with centre O, two tangents PQ and PR are drawn such that $\angle QPR = 120°$, prove that 2PQ = PO.

330. Two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.

331. The incircle of ΔABC touches the side AB, BC and AC at P, Q and R respectively. If AB = 8cm, BC = 10cm and AC = 12cm. Find the length of AP, BQ and CR.

332. The radii of two concentric circles are 13cm, and 8cm. AB is a diameter of a bigger circle and BD is tangent to the smaller circle touching at point D. Find the length of AD.

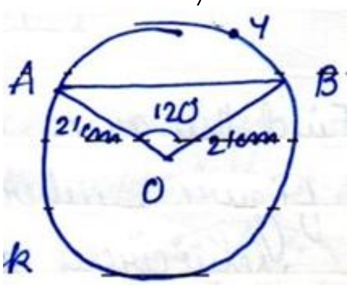
333. Two concentric circles are of radii 7cm and r cm respectively where r > 7. A chord of a larger circle of length 46cm touches the smaller circle. Find the value of r.

AREAS RELATED TO CIRCLE

334. The cost of fencing a circular field at the rate of `24 per metre is `5280. The field is to be ploughed at the rate of `0.50 per m^2. Find the cost of ploughing the field [take $\pi=\frac{22}{7}$]

335. The wheel of a car are of diameter 80 cm each. How many complete revolutions does each wheel make in 10 minutes when the car is travelling at a speed of 66 km per hour?

336. Find the area of the segment AYB shown in the figure, if radius of the circle is 21 cm and $\angle AOB = 120^0$ [use $\pi=\frac{22}{7}$]



337. The length of the minute hand of a clock is 14 cm. find the area swept by the minutes hand in 5 minutes.

338. A chord of a circle of radius 10 cm subtends a right angle at the centre. Find the area of the corresponding

i) minor segment ii) major segment [use $\pi = \frac{22}{7}$]

339. In a circle of radius 10 cm subtend a right of 60^0 at the centre. Find

i) the length of the arc ii) area of the sector formed by the arc

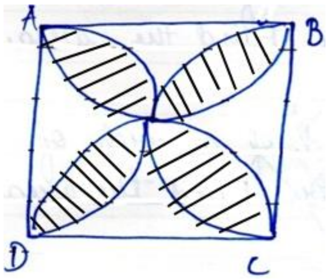
iii) area of the segment formed by the corresponding chord.

340. A brooch is made with silver wire in the form of a circle with diameter 35 mm. the wire is also used in making 5 diameter which divide the circle into 10 equal sectors as shown in the figure. Find:

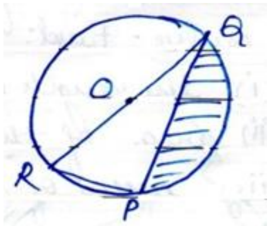
i) the total length of the silver wire required ii) the area of each sector of the brooch.

341. A round table cover has six equal designs as shown in the figure. If the radius of the cover is 28 cm, find the cost of making the designs at the rate of `0.35 per cm^2 [use $\sqrt{3}$=1.7]

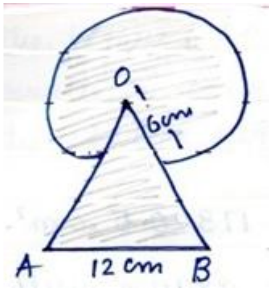
342. Find the area of the shaded design in figure, where ABCD is a square of side 10 cm and semicircles are drawn with each side of the square as diameter. [use π = 3.14]



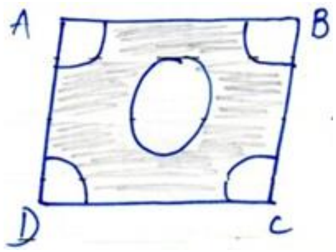
343. Find the area of the shaded region in figure, if PQ = 24 cm, PR = 7 cm and O is the centre of the circle.



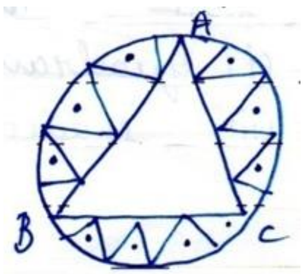
344. Find the area of the shaded region in the figure, where a circular arc of radius 6 cm has been drawn with vertex O of an equilateral triangle OAB of side 12 cm as centre.



345. From each corner of a square of side 4 cm a quadrant of a circle of radius 1 cm is cut and also a circle of diameter 2 cm is cut as shown in the figure. Find the area of the remaining portion of the square.



346. In a circular table cover of radius 32 cm, a design is formed leaving an equilateral triangle ABC in the middle as shown in figure. Find the area of the design.

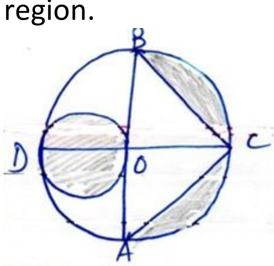


347. The figure depicts a racing track whose left and right end are semi circular.

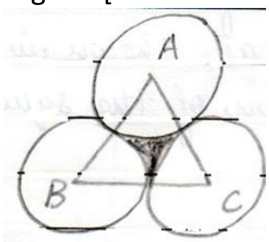
The distance between the two inner parallel line segments is 60 m and they are each 106 m long. If the track is 10 m wide, find:

i) the distance around the track along its inner edge ii) the area of the track

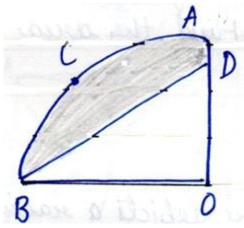
348. In the figure, AB and CD are two diameters of a circle (with centre O) perpendicular to each other and OD is the diameter of the smaller circle. If OA = 7 cm, find the area of the shaded region.



349. The area of an equilateral ΔABC is 17320.5 cm^2, with each vertex of the Δ as centre, a circle is drawn with radius equal to half the length of the side of the Δ. Find the area of the shaded region. [use π = 3.14 and $\sqrt{3}$ = 1.73205]



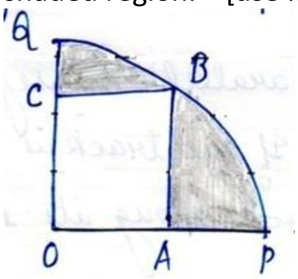
350. In the figure, OACB is a quadrant of a circle with centre O and radius 3.5 cm. if OD 2 cm, find the area of the



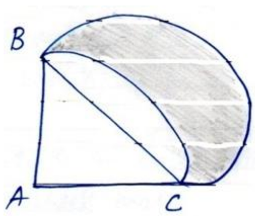
i) quadrant OACB.

ii) shaded region.

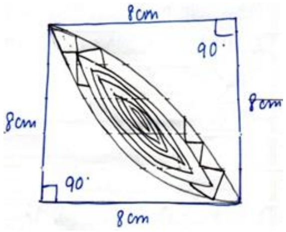
351. In the figure, a square OABC is inserted in a quadrant OPBQ. If OA = 20 cm, find the area of the shaded region. [use π = 3.14]



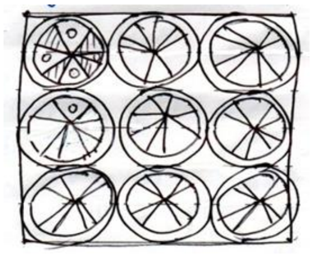
352. In the figure, ABC is a quadrant of a circle of radius 14 cm and a semicircle is drawn with BC as diameter. Find the area of the shaded region.



353. Calculate the area of the designed region in the figure common between the two quadrants of circles of radius 8 cm each.



354. On a square handkerchief nine circular designs each of radius 7 cm are made (see the figure). Find the area of the remaining portion of the handkerchief.



STATISTICS

355. Find the mean of the following table by direct method:

Class interval	0 –10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	8	12	10	11	9

356. The arithmetic mean of the following data is 25 determine the value of P.

Class interval	0 –10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	18	15	P	6

357. If the mean of the following frequency distribution is 65.5, find the missing frequencies $f1$ and $f2$.

Class interval	10 –30	30 – 50	50 – 70	70 – 90	90 – 110	110 – 130	Total
Frequency	5	8	$f1$	20	$f2$	2	50

358. Fin the mean marks per student using assumed mean method and step deviation method.

Marks	0 –10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	12	18	27	20	17	6

359. The following table shows the marks scored by 80 students in an examination. Find mean.

Marks	less than 5	less than 10	less than 15	less than 20	less than 25	less than 30	less than 35	less than 40
No. of students	3	10	25	49	65	73	78	80

360. Find the median wage from the following data.

Wages in Rs.	800 – 820	820 – 840	840 – 860	860 – 880	880 – 900	900 – 920	920 – 940
No. of workers	7	14	19	25	20	10	5

361. Calculate the median for the following data:

Marks obtained	No. of students
below 10	6
below 20	15
below 30	29
below 40	41
below 50	60
below 60	70

362. Find the missing frequencies in the following frequency distribution table if N = 100 and median is 32.

Marks	0 –10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	Total

No. of students	10	?	25	30	?	10	100

363. Calculate the missing frequency from the following distribution, it being given that the median of the distribution is 24.

Class interval	0 –10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	25	?	18	7

364. Find the median wages for the following frequency distribution.

Wages / Rs.	61 – 70	71 – 80	81 – 90	91 – 100	101 – 110	111 – 120
No. of workers	5	15	20	30	20	8

365. Find the mode of the following data:

Class interval	0 –20	20 – 40	40 – 60	60 – 80	80 – 100	100 – 120	120 – 140
Frequency	6	8	10	12	6	5	3

366. The mode of the following series is 36, find the missing frequency in it.

C.I.	0 –10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	8	10	….	16	12	6	7

367. The following table shows the marks obtained by 100 students of class X in a school during a particular academic session, find the mode of the distribution.

Marks	less than 10	less than 20	less than 30	less than 40	less than 50	less than 60	less than 70	less than 80
No. of students	7	21	34	46	66	77	92	100

368. Find the mean, mode and median of the following frequency distribution.

Class interval	0 –10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	18	15	12	6

369. For the following frequency distribution, draw a cumulative frequency curve of more than type and hence obtain the median value.

C.I.	0 –10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	5	15	20	23	17	11	9

370. The following are the ages of 300 patients getting medical treatment in a hospital on a particular day.

Age in years	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
No. of patients	60	42	55	70	53	20

Form a 'less than type' cumulative frequency distribution.

371. The table given below shows the weekly expenditures on food of some households in a locality.

Weekly expenditure in Rs.	Number of households
100 – 200	5
200 – 300	6
300 – 400	11
400 – 500	13
500 – 600	5
600 – 700	4
700 – 800	3
800 – 900	2

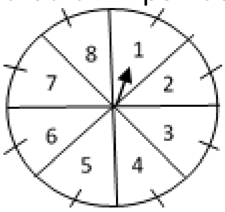
Draw a 'less than type ogive' and 'more than type ogive' for the distribution on the same graph end hence find the median.

PROBABILITY

372. If P(E) = 0.05, what is the probability of 'not E'?

373. A box contains 5 red marbles, 8 white marbles and 4 green marbles. One marble is taken out of the box at random. What is the probability that the marble taken out will be (i) red? (ii) white? (iii) not green?

374. A game of chance consists of spinning an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 (see fig.) and these are equally likely outcomes. What is the probability that it will point at:



(i) 8? (ii) an odd number?
(iii) a number greater than 2? (iv) a number less than 9?

375. A die is thrown once. Find the probability of getting
(i) a prime number, (ii) a number lying between 2 and 6. (iii) an odd number

376. One card is drawn from a well-shuffled deck of 52 cards. Find the probability of getting
(i) a king of red colour (ii) a face card (iii) a red face card
(iv) the jack of hearts (v) a spade (vi) the queen of diamonds

377. Five cards – the ten, jack, queen, king and ace of diamonds, are well-shuffled with their face downwards. One card is then picked up at random. (i) What is the probability that the card is the queen? (ii) If the queen is drawn and put aside, what is the probability that the second card picked up is:
(a) an ace? (b) a queen?

378. (i) A lot of 20 bulbs contain 4 defective ones. One bulb is drawn at random from the lot. What is the probability that this bulb is defective?
(ii) Suppose the bulb drawn in (a) is not defective and is not replaced. Now one bulb is drawn at random from the rest. What is the probability that this bulb is not defective?

379. A box contains 90 discs which are numbered from 1 to 90. If one disc is drawn at random from the box, find the probability that it bears (i) a two-digit number (ii) a perfect square number (iii) a number divisible by 5.

380. A die is thrown twice. What is the probability that
(i) 5 will not come up either time? (ii) 5 will come up at least once?

381. A bag contains 5 red balls and some blue balls. If the probability of drawing a blue ball is double that of a red ball, determine the number of blue balls in the bag.

382. A box contains 12 balls out of which x are black. If one ball is drawn at random from the box, what is the probability that it will be a black ball?

383. A jar contains 24 marbles, some are green and others are blue. If a marble is drawn at random from the jar, the probability that it is green is $\frac{2}{3}$. Find the number of blue balls in the jar.

********************END********************